

Research Article

# Concept cathedral and “squaring the circle”: Interpreting the Gothic cathedral of Notre Dame de Paris as a standing hymn

Nelly Shafik Ramzy

Department of Architectural Engineering, Faculty of Engineering, Benha University, El Kalyobia, Egypt

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**Abstract** Semiotics is used in the sphere of visual arts as a tool that helps *reading the message* included in a certain work. Gothic architecture, as an outcome of the rich cultural context of medieval tradition, where metaphysics, numbers and geometry were indispensable tools to symbolically manifest cosmological and theological concepts, is suggested as a good example for testing this approach in the field of architecture.

In this paper, a semiotic reading for one of the most distinguished works of this era; i.e., the cathedral of Notre Dame de Paris, is introduced as a model for this methodology. Among many message included in the building, such as the functional or the technical messages, this is a trial to introduce a reading, i.e., an interpretation, of what could be called “*a hidden spiritual message*”. The paper comes to a conclusion that the Gothic cathedral, as represented in Notre Dame de Paris, might be perceived as an externalization of a *standing hymn* that was meant to send continuous appeals of protection to Heaven.

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## 1. Introduction

Ernst Cassirer comprehensively classifies man as a *symbolic animal*, with the very structure of the human mind constituted to perceive in a symbolic way. Accordingly,

meaning becomes a necessary, fundamental, and immanent requirement of human sanity (Cassirer, 1955).

Cities are chronicles in stone and architecture is described as “giant alphabet” and “the biggest system of symbols” (Chesterton, 1920). Lines, planes, colors, textures are “the syntax and grammar” of this language (Barabanov, 2002). Well into the 19th century, the educated viewer could read a building as one reads a text, but today this language is largely mute to viewers, as much of its meaning lost and eroded by time (Rimmer, 1997).

E-mail address: [tawswzwm@yahoo.com](mailto:tawswzwm@yahoo.com).

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Quite on contrary to the city of today, medieval buildings were built up out of what the soul felt and thought, moving among them, "the soul forces streamed over to people" (Steiner, R., 2001), where, according to Nigel Hiscock, "for medieval man, the search for inner meaning was a habit" (Hiscock, 2007). This perspective encompasses what Richard Utz calls "Concept Cathedral" (Utz, 2005), where art and geometry were used as means to express ideas of religious and philosophical nature. At this time, God was thought to be the *Geometer*, as shown in the medieval famous painting, where the compass was seen as a symbol of God's act of *Creation*.

Above all, it needs to be understood that, until the late Middle Ages, churches were designed not for the eyes of human beings, but for the eyes of the Divine, who was invited to dwell in these buildings (Dudley, 2001). Masons of the Gothic cathedrals employed esoteric knowledge and aesthetic expressiveness to produce forms and patterns that can be enjoyed by the masses, whilst preserving an inner meaning for those, who can perceive it, and in the first place for the eye of the Divine (Gavins, 2005). This did not mean that every craftsman had to be a theologian; his work would still be influenced by the shared ideas of the theologians. Robert Scott explains that master masons, who oversaw the building projects, were all educated in religious schools, i.e., were well versed in theological concepts and eager to implement them (Scott, 2003).

The most fundamental concern that faced both bishops and masons of this era was how to make their cathedrals a replica of the *Heavenly Jerusalem*, so that they become worthy of God's presence (Brümmer, 2008). Another concern was how to protect these cathedrals from disasters and evil forces. This paper illustrates how medieval masons worked out these concerns by including a system of signs into the designs of their cathedrals.

The fire which struck *Notre Dame de Paris* in April of 2019 drew attention to one of the world's remarkable pieces of sacred architecture. The paper employs semiotic tools of analysis to undertake an interpretation of the architectural language in this masterpiece to demonstrate how figurative, numerical, and geometrical semiotics were employed to connote the concept of *squaring the circle* and to make possible a reference message that comprises an *everlasting standing prayer*.

In this concern **the paper aims at** (a) providing a model, in which the architectural language in the cathedral can be read as a text; (b) testing the semiotic analyses against the "secret language" of the medieval masons; and finally (c) exploring the building's formal expression from semiotic perspective to recognize how to *weave* our own legacies into our contemporary architecture. **In conclusion**, a full reading, i.e., interpretation, of the architectural text of the cathedral, as an everlasting standing prayer, is introduced, suggesting that the semiotic analysis is a valid tool to decode the *secret language* of the medieval masons and identify the patterns of meaning-construction in their works.

**The paper is structured as follows:** methodology, where the tools of semiotic analyses that will be used to decode/interpret the architectural language in the building are overviewed; theoretical background, where the concept of squaring the circle in medieval tradition is explained; analysis, where a semiotic analysis of the forms, structures and organization of the selected case study is performed;

discussion, where the results of the analyses are to be discussed; and finally conclusion, where the findings of the research are to be summarized.

## 2. Methodology

Semiotics is the study of signs. It involves the study of any medium as a *sign system* and semioticians commonly refer to all kinds of medium as *texts*. It was initially applied to language; but theorists like Barthes, Greimas and Eco extended these ideas to all sorts of visual arts including architecture. The interpretation of such texts is always subjective and/or arbitrary; therefore the same *architectural text* may have more than one *reading* (Juodintytė-Kuznetsova, 2011).

Notre Dame de Paris is the most famous of all Gothic buildings. Key elements of Gothic style are identified in this cathedral that it has been referred to as "the world ambassador of Gothic Cathedrals" (Bony, 1983). Therefore, this cathedral had been chosen to be analyzed in this paper as a representation of Gothic style. In order to deal with this *text*, **the following steps were followed:** (a) a purposive study of the different models of semiology and their applications in visual communication (b) a contextual study for the selected cathedral; (c) an extensive reading in the dominant theological traditions of medieval era; (d) a component analysis of the elements, structures and organization of the architectural form of the cathedral; (e) an adoption of a logical structure of the signs system within the cathedral.

Signs must be interpreted according to certain conventions, which semioticians refer to as *codes*, which represent the socio-cultural dimension in semiotics (Fiske, 1989).

Concerning the cathedral of Notre Dame de Paris, the wide framework of the medieval theology and tradition is the code for interpreting this text. A more specific code is yet to be found in the Neo-Platonic theories, which were incorporated into the Christian theology by the writings of St. Augustine, giving wide range of interpretations for numerical, figurative, and geometrical symbols in relation to Christian theology. St. Augustine's interpretations to numerical symbols, for example, ranged from simple reference to biblical number/events or sacred proportions of certain structures, like Solomon's temple, to a detailed manipulation of numbers, i.e., breaking down numbers into their factors or reference to psalms' numbers (Augustine, 1972).

The semiotic analysis, which was applied onto Notre Dame de Paris in this paper as a model for reading *architectural texts*, comprises the following steps: (a) identifying the signs in the building (iconic, symbolic, and indexical); (b) interpreting them according to the code (signifier/signified); (c) classifying them into: denotation and connotation, metaphor and metonym; and (d) arranging them into paradigms and syntagms to understand the full text.<sup>1</sup>

<sup>1</sup> See N. Ramzy, "The visual language in Mamluke Architecture: A semiotic analysis of the Funerary Complex of Sultan Qaitbay in Cairo". *Frontiers of Architectural Research*, 2 (3): 338–353 for a full review of the elements of Semiotic analysis as tools for analyzing architectural works.

### 3. Theoretical background

According to Hegel, "in the Middle Ages art's, function was to a large degree to make the divine visible" (Hegel, 1975), where, according to St. Augustine, "Only through faith in God, the architect of the world, the invisible things of God are seen" (Schaefer, 2009).

At this time, the School of Chartres was the most prominent theological school in Paris. Their teachings were strongly influenced by the Neo-Platonism, as it was represented by St. Augustine, who said that: "There is none who comes nearer to us than the Platonists" (Augustine, 1972). Abbot Suger, who is credited to be the founder of Gothic style, was one of the followers of this school, and it was through this school that the Neo-Platonism of St. Augustine dominated the designs of Gothic cathedrals in France (Bandmann, 1951).

In the teachings of St. Augustinian: "the purpose of geometry was to sanctify the building by making it part of the heavenly world of the God" (Dudley, 2001). Documents of the period, such as Mathes Roriczer's (1492) and Hanns Schmuttermayer's (1487) booklets, tell much about how this "divine activity" (Clark, 1969) was actively theological and was employed to sanctify religious buildings and miraculously preserve them from destruction. The main purpose of architectural design was to produce a building that would unite with the eternal world of Heaven and thus to be protected from disasters and evil forces (Dudley, 2001). The *spiritual design* of medieval cathedrals was therefore governed by two rules; both contribute to the concept of uniting the building with the heavenly geometry:

The first rule is Euclid's *Ad Quadratum*, or square in circle, which was developed by theologian of the era into *squaring the circle* (Hendrix, 2003), making an analogy, in which the square represents limited space of Earth, whereas the circle, which has no beginning and no end and is the source-figure, from which all other geometrical shapes emerge, represents the boundlessness of the Heavens (Lundy, 2001).

It was first in Ancient Egypt that a belief in this geometrical concept being capable of creating a link between ephemeral and eternal lives occurred and it was believed that **the great pyramid of Cheops** geometrically united Heaven and Earth. Since then, the Pyramid together with its angles and proportions were seen an icon of *squaring the circle* (Reynolds, 1999). Further applications of this concept include: the *Vesica Piscis*, the *Sacred Cut* and the *Pentagram*.

The second rule was **integrity with the geometry of Nature**. This rule had its origins in the Augustinian belief that the universe owes its stability to the perfect balance of its elements as instituted by the Creator, a stability that will be denied to any religious building that does not follow the same geometry (Dudley, 2001). It was essential then to the geometry of a building to be integrated with the geometry of the universe and acts as an extension to it. **Geometric progressions** were believed to encompass the characteristics of natural growth as well as the power of "permanence"; unlike arithmetical progressions, it is a property of geometric progressions that they can extend in

either direction, larger or smaller, to an infinite degree, to the dome of Heaven and beyond, but it can never become a minus quantity or reduce to zero. So, they were deeply believed to have the power to provide security from the destructive forces. Further applications of the Geometry of Nature include: **irrational numbers**, **fractals** and **Golden Section**.

#### 3.1. Irrational numbers

St. Augustine found numbers and geometrical forms to be imbued with meaning that he considered numbers as "the thoughts of God" (Mâle, 1961). In addition to simple whole number ratios, *Ad Quadratum* creates those strange ratios that are known as irrational numbers, or surds, which cannot be accurately expressed numerically such as  $\sqrt{2}$ ,  $\sqrt{3}$  and  $\sqrt{5}$ , but easily produced as geometric ratios.<sup>2</sup> They were thought to be comprehended only by the Creator of the cosmos and had therefore fundamental role in sacred geometry of the Universe. The most mysterious ratio, however, was the  $\pi$ , which cannot be created geometrically except by drawing a circle. Such a number was called 'transcendental' and was regarded as truly divine (Gracia, 1992).

#### 3.2. Golden Ratio

The **Golden Ratio**, also called the "divine proportion" (Summerson, 1963), or  $\phi$ , is another *irrational* number. Its significance is that it portrays the manner, in which many things grow in Nature, it is also the base of geometrical progression (Fibonacci sequence), and it has a strong presence in *squaring the circle* (Naredi-Rainer, 1982) (Fig. 1).

In his 1919-book *Ad Quadratum*, Lund claims that several medieval cathedrals, including Notre Dame de Paris, are designed according to the Golden Ratio (Lund, 1919), where Prof. R. Bork also suggests that the facade of Strasbourg cathedral was designed according to this ratio (Bork, 2011).

#### 3.3. Vesica Piscis

**Vesica Piscis** was one of the key starting blocks in the design of Gothic cathedrals as illustrated in Cesare Cesariano's 1521-book *"The Rule of the German Architects"*. In addition to the  $\pi$  in its structure, the surds, i.e.,  $\sqrt{2}$ ,  $\sqrt{3}$ , and  $\sqrt{5}$ , as well as the Golden Ratio, are all present within the realm of its overlap (Dabbour, 2012) (Fig. 2).

The *Vesica Piscis* (literally means *fish bladder*) has many connotations in Christian theology: fish's eyes never close, symbolizing God's unceasing watchfulness over his church (Ramzy, 2015); *Fish* in Greek is ΙΧΘΥΣ, which are the initial letters of Iesous Xristos Theou Yios Soter (Jesus Christ, Gods Son Savior); and it is a symbol for the intersection of Heaven and Earth or Christ's double nature as God and Man (Ramzy, 2014).

<sup>2</sup> The square includes  $\sqrt{2}$ , the hexagon includes  $\sqrt{3}$ , the circle includes  $\pi$  and the pentagon includes the Golden Section.

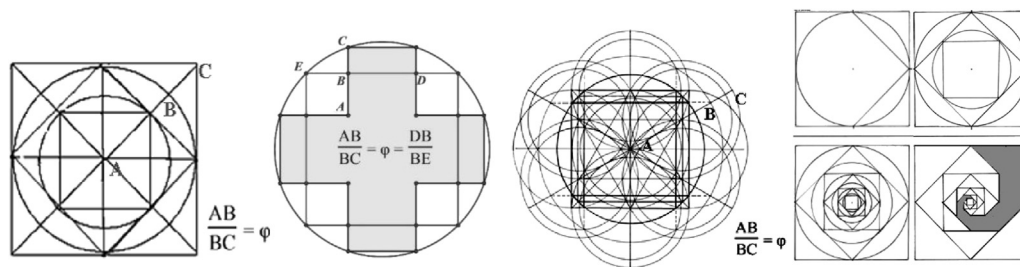


Fig. 1 The Golden Section and squaring the circle.

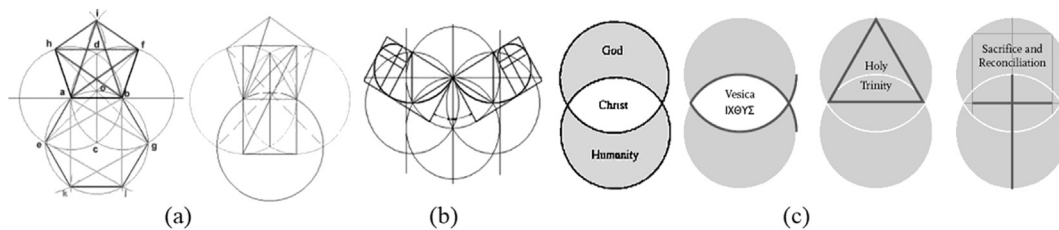


Fig. 2 (a) Square roots within the *Vesica Piscis* (Based on De l'Orme (1567)), (b) the Golden Ratio nested in the *Vesica Piscis*, and (c) *The theological Vesica Piscis* (based on Megan Hitchens (1996)).

One further aspect that occurs in the *Vesica Piscis* is that by drawing two orthogonal *Vesicas*, then connecting the four intersection points of the *Vesica* to get a square. This square is approximately equal in perimeter to the original circle, i.e., a *squared circle* (Critchlow, 2000) (Fig. 3a).

### 3.4. Sacred cut

The traditional method of constructing an octagon begins with the so called *Sacred Cut* (Fig. 3b). Brunes claims that this figure was fundamental to the construction of monuments from the Egyptian to the Medieval. The *sacredness* of this figure lies in its very nearly solving the riddle of how to *square the circle* (Brunes, 1967).

### 3.5. Pentagon

In *The Elements*, Euclid introduced the regular **pentagon** as the most remarkable application of the *Extreme Ratio*, or the Golden Ratio, as its diagonals cut each other at this ratio (Livio, 2003). During the medieval era, the pentagram star was seen as the symbol of mankind (Child and Colles,

1971). The man and the pentagram were symbols of *stigmata*, which is the symbol of God's protection to his people. It is also a symbol of man inside a polygon and a circle (Pineda, 2011).

The pentagram's special relationship to the Golden Section represents a possibility of infinite divisions in both directions, and thus augments the mystic connotations of this figure to continuity and eternal life (Fig. 4).

### 3.6. Fractals

Patterns that exhibit infinite self-similarity are very common in Gothic architecture (Goldberger, 1996), where details such as pinnacles echo the forms of larger elements such as spires. These patterns are now known as *fractals*.<sup>3</sup> Geometrical progression, together with the tendency to follow the natural forms of growth, were the obvious links that led Gothic artists to this type of patterns, in which, these two *protective qualities* are leading characteristics, despite, at this time, any idea about the principles of fractal geometry, as they are known today, was still completely missing (Ramzy, 2015).

## 4. Analysis

Construction of Notre Dame De Paris started in 1163 according to a design by Bishop *Maurice de Sully* and was completed in 1345.<sup>4</sup> It was the first cathedral to be built on a monumental scale and became the prototype for future cathedrals in France, comprising a group of cathedrals,

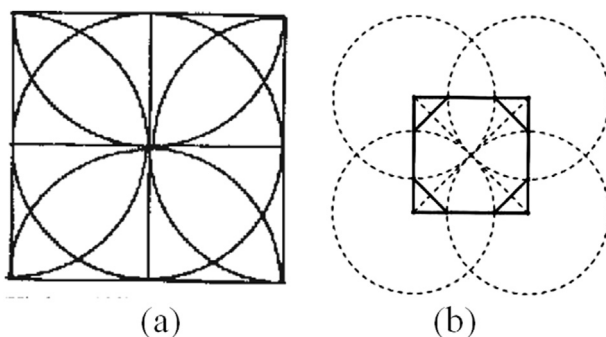
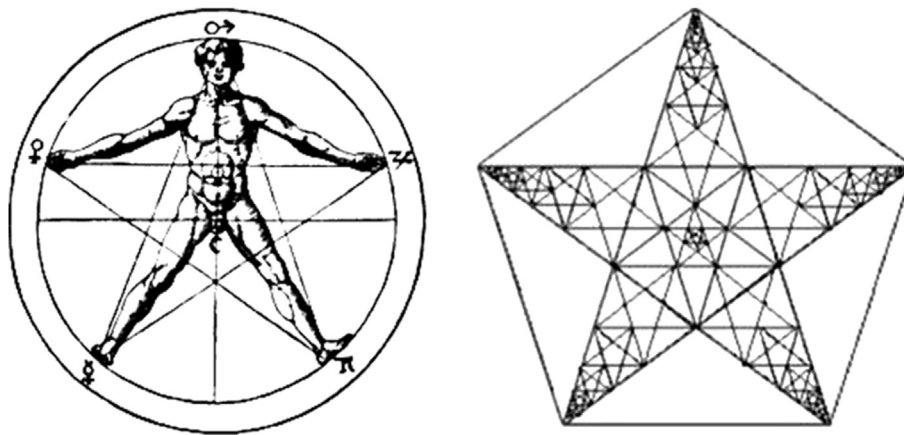


Fig. 3 (a) The squared circle and (b) the sacred cut.

<sup>3</sup> A fractal is a pattern that repeats itself at different scales to an infinitely small scale (like cauliflower).

<sup>4</sup> Eugène Emmanuel Viollet-le-Duc carried out extensive renovations for the cathedral between 1845 and 1864.





**Fig. 4** (left) Agrippa's Man: Man Inscribed in the Pentagram (public domain), (right) the Pentagrams exhibit self-symmetry and infinite fractal expansion by use of Golden Section.

including Amiens, Chartres, Strasbourg and Rheims, all dedicated to *Notre Dame*. The physical locations of these cathedrals is said to form a pattern that venerate the star constellation of Virgo (Gardner, 1996).

In the following is a *reading* of the semiotic system of this cathedral, in a trial to decode its architectural *text*. It also shows how the previously mentioned applications of *squaring the circle* are all strongly present in the geometric design of the cathedral.

#### 4.1. Plan

As a rule, the foundation of the Gothic cathedral used to begin with the orientation of the axes towards the East. Twelve stones for the foundations were washed with water and wine in reference to the Last Supper. The main site was then marked with a circular base, the center of the future cathedral was established, and the four corners of the ground square marked by four stones, signifying the four corners of the world (Conybeare, 1905) and symbolizing the stability of the square as the most stable geometrical figure, that "stands firm on any side" (Hiscock, 2007). The, always cruciform, ground plan was then laid and the construction went then from north to south at each level (Bruzelius, 1987).

The axis of the nave at Notre dame de Paris is  $26^\circ$ , indicating the sun-rise at winter solstice in France, while the axis for the choir is  $23.5^\circ$ , which is the precession angle for the rotation of the Earth. The relationship between the two axes connotes the connection between Heaven and Earth (Benishai, 2017).

Despite it was the West End that functioned as the main entrance, construction always began at the East End of the cathedral (De Honnecourt, 1953). The construction process of Notre Dame de Paris was divided into seven stages (Bruzelius, 1987): The *chevet*, the chancel, the transept, the northern aisles, the southern aisles, the nave, and finally the portals. Seven is a number that is closely associated to rituals and prayers in Christian theology: seven prayers in the day, seven requests in the Pater Noster, seven gifts of the Holy Spirit, etc. Horn and Born (1975). It is also the number of colors in the rainbow (a symbol of

connection between Heaven and Earth). St. Augustine said that it expresses completeness (the seventh day of creation) and the sum of four (Earth) and of three (Holy Trinity). It refers also to the seven days of the week, and man's life span consists of periods of seven years: from birth to the change of teeth; puberty; adulthood, etc. (Horn and Born, 1975).

The nave of the cathedral is also divided into seven bays in both directions; vertical and horizontal, supported by seven flying buttresses on each side like the oars of a giant Ark. The total number of the bays at the cathedral's plan is 153 (portals 3, narthex 15, nave 49, transept 7, choir 35, and chevet 44). This is the number of fish that were caught by the disciples, when Jesus advised them to put their net on the other side of the boat. This number came to represent the whole Church of Christ (Bannister, 1968).

St Augustine linked the ratios of Noah's ark to the perfect human form of Christ. In Notre Dame of Paris, the main level is thirty cubits high, the same height as the Ark of Noah and Solomon's Temple. The overwhelming building, including the projection of the flying buttresses, is 132 m long (closed to that of Noah's Ark (137 m).

The most important characteristic of Gothic plans was the apsidal termination of the choir, or *chevet*, with radiating chapels that were not divided into separate compartments, but were unified. The chevet here contains thirteen chapels; 6 on each side and one main chapel with an apse at the middle (Christ unified with the twelve apostles and the twelve tribes of Israel (people of God)). The small chapels on the sides of the choir and the nave are also twelve for each of them. This number is the basis of the astronomical systems: the 12 h of the day/night, the twelve months of the year, the twelve divisions of the zodiac, etc. (Horn and Born, 1975). Neo-Platonic theologians believed that it was for profound mathematical reasons that Christ chose twelve as the number of his disciples. It is the product of three (the Trinity) and four (the four corners of the word and the four elements of matter), and thus in the twelve, matter is infused with divine and the Universal Church is established (Mâle, 1961).

The number of columns on each side of the nave, as well as on the sides of the chancel, is eight, which is the number

of notes in the tonal scale and the number of semi-consonants in Latin language, also related to the musical theory (Chevalier and Gheerbrant, 1988). The total number of piers in the cathedral is 26, also corresponding to the total number of letters in Latin language.

The plan has five main divisions (chevet, choir, transept, nave/aisles and narthex). Five is a number that had double connotation in Neo-Platonic theology. It connotes the *stigmata*; a symbol of protection (Pineda, 2011), while its geometric symbol, the pentagram, is a symbol of divine proportions (human body and Golden Section). It is also the sum of  $4 + 1$  (the Matter and the Devine) (Hopper, 1938).

The plan widely employs the geometric principles of Ad Quadratum, Vesica Piscis, Sacred Cut and pentagram, to determine the intersecting key points (Fig. 5).

In describing the geometric design of this floor plan, R. Bork says that it is dependent on a “dynamically unfolding geometrical operations” (Bork, 2014), in which “geometry provides a quasi-random growth factor”<sup>5</sup> in reference to everlasting growth.

Several medieval symbols can be also realized by analyzing the key points of the plan (Fig. 6). The overall ground plan is based on a Vesica Piscis containing two pentagrams, the points of which correspond to the so called *Light Matrix*, or, *Ain Sof*, where Spirit (light) and Matter (man) were believed to come together (Ramzy, 2015).

The *Sephiroth* or the *Tree of Life* is another symbolic figure that can be recognized in the plan. In medieval literature this symbol was believed to have a divine power (Endres and Schimmel, 1984; Horn and Born, 1975). The proportions of human body (divine proportions) are also present in the design of the plan (Jencks, 2013).

## 4.2. Exterior

The fronts of Notre Dame de Paris are known for their sublime, balanced appearance that is supplemented by sculptures of people and angels scattered all over the fronts in the position of devoutness and reverence as if in prayers. In the following is a *reading* for these fronts:

## 4.3. The west façade

The west front is an imposing, simple mass, whose grandeur is based on interplay between vertical and horizontal lines (Heaven and Earth): four powerful buttresses spring up to the top of the towers, lifting them heavenwards, while two wide horizontal strips (Gallery of kings<sup>6</sup> and *Galerie des Chimères*) seem to bring the building back down to mortal Earth, dividing the squared base of the façade into four *realms*. In Neo-Platonic theology, four is a number that has double connotation of both celestial and material worlds; it is the number of Realms (Matter, Nature, Celestial, and Super-celestial) and it represented the four Gospels that spread the faith to the four corners of Earth. It is also the sign of Earth with its four cardinal points, four elements,

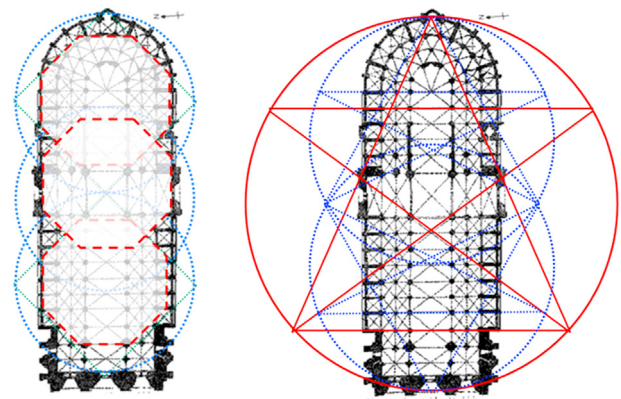


Fig. 5 Ad Quadratum, Vesica Piscis, sacred cut and pentagram in the plan of Notre Dame de Paris.

four states of matter, four winds, and four seasons (Buckland (1896); Horn and Born (1975)).

The rows of vertical elements, of which the horizontal strips consist (row of portals, row of statues, row of pointed arches), are like rows of arrows that point heavenwards.

The dimensions of the façade, up to the top of the *Galerie des Chimères*, are of a square 141 ft × 141 ft. Psalm 141 is the psalm of “Lord, I have cried to you, hear me”, which St. Augustine calls: “Passion of the whole body of Christ” i.e. the Church (Augustine, 1972). The last three verses of which are all about protection and survival.

By overlaying the scheme of the west façade over that of the ground plan, a significant correspondence between them can be seen, in reference to the correspondence between horizontal and vertical levels (Heaven and Earth). This façade is one of the most renowned examples of the application of Golden Ratio (Fig. 7). Le Corbusier described it as a “pure creation of the spirit, ...governed by the circle and the square” (OSNDP, 2020.), and thereby is also the plan.

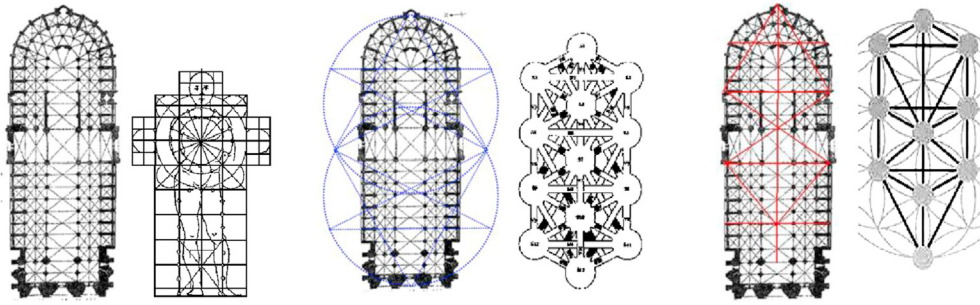
Vesica Piscis is also a dominant feature in the geometry of this façade. The overall mass of the façade could be included in vertical Vesica Piscis; one circle descending from Heaven and the other rising from Earth (Schneider, 1995). The portals also have similar geometric structure, but the two circles of the doorway are at horizontal position (Fig. 7).

Jonathan Hale notes that the angle at the base of the triangle that links the Rose Window and the portals is  $52^\circ$ ; very close to that of the Pyramid of Cheops ( $51^\circ 51'$ ) (Hale, 1995). Another “pyramid” is to be found by linking the ends of the *Galerie des Chimères* with the point at the middle of the horizontal line between the two towers (Fig. 8). He says that the Pyramid here expresses “certain spiritual qualities, as the earliest and most well-known symbol of *squaring the circle*”, where Joseph Campbell says that at the top of the pyramidal shape “the eye of God opens” (Campbell, 1988). The inclination of the inner timber frame of the roof is also at  $52^\circ$  (Bruzelijs, 1987).

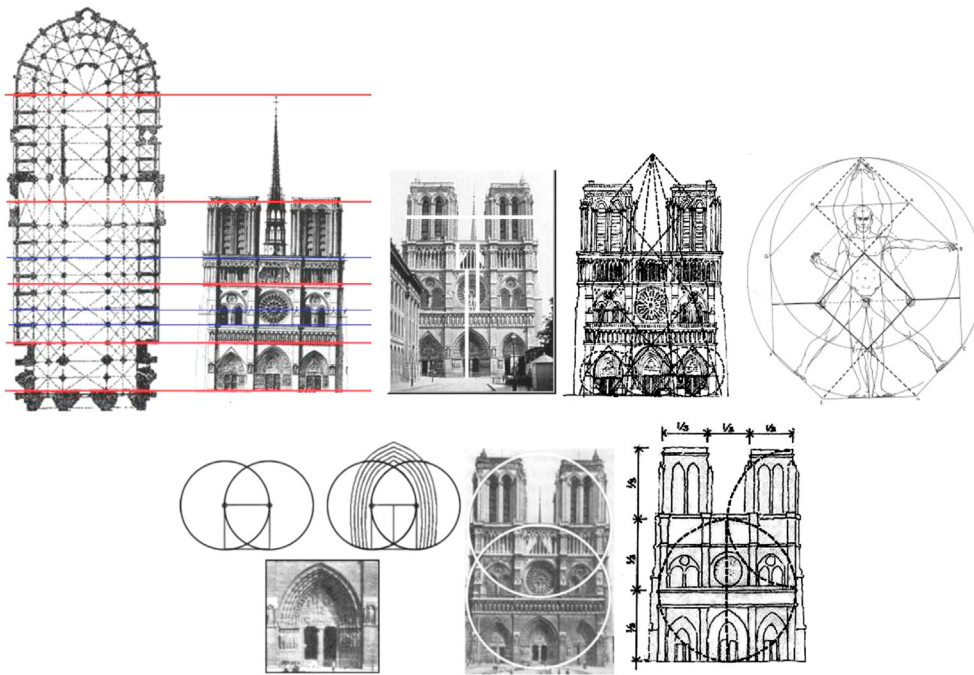
As in plan, the façade is divided into five gigantic sections (*stigmata* - divine proportions - Earth + the Devine). The first thing to see in this façade is the **three portals**.

<sup>5</sup> See Bork (2014) for further demonstration for the geometric analyses of the Chevet at Notre-Dame in Paris.

<sup>6</sup> A line of statues of the 28 Kings of the Old Testament.



**Fig. 6** Plan of Notre Dame de Paris and its relationship to human body, the *Light-Matrix*, and the *Tree of Life* (both related to the pentagram and the Vesica Piscis).



**Fig. 7** Geometric analysis of the west façade: (above, from left to right) the dimensions of plan contained in the elevation, the Golden Section, the Ad Quadratum, and the proportions of human body (Neufert, 1936) in the design of the façade, (below) the Vesica Piscis in the design of the façade.

Their pointed arches are the first symbol of ascension in the façade.

The general theme of the portals is The Gate to Heaven (OSNDP, 2020). The left portal is The Portal of the Virgin and it depicts her death, her ascension into Heaven and her coronation as Queen of the Heavens in front of the Heavenly Court (angels, patriarchs, kings, martyrs and prophets) on four of the five successive archivolts of the portal. In addition to the above mentioned connotations, five also has a strong relationship with appealing to god (five types of offerings in the Old Testament; the five divisions of the Book of Psalms,<sup>7</sup> and in Psalm 5 David asks the Lord to hear his voice. It is also a reference to the five wise women). The

abutments of the doors have panels representing the natural/annual cycles (Zodiacs and Labors of the Months).<sup>8</sup> On the jambs are the seasons (left) and the ages of man (right).

The Portal of Saint Anne is similar to the Portal of the Virgin. At the center of it is the Virgin seated on a throne, with an angel on each side of the throne, the bishop of Paris on the left, the king of France on the right and figures of the Heavenly Court on the concentric archivolts.

On the lower lintel of the middle Portal of the Last Judgment, the dead are being resuscitated from their tombs. On the upper lintel, the archangel Michael is weighing the souls and leading them either to Heaven or to Hell. On the tympanum, Christ is seated on His throne of

<sup>7</sup> (1–41) the Passover and salvation; (42–72) unified Israel and New Jerusalem; (73–89) destruction of Jerusalem; (90–106) the reign of Christ; (107–150) Judah shall again be delivered.

<sup>8</sup> The positions of the months echo the yearly cycle of the sun: rising in the sky from January to June (left jamb), then descending from July to December (right jamb).



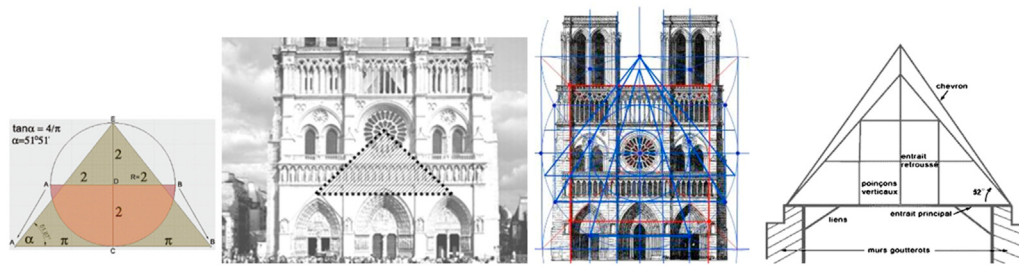


Fig. 8 The angles of the pyramid and squaring the circle.

glory. Like on the other portals, the archivolts feature the Heavenly Court, but the archivolt here has 7 recesses (the number of prayers). Psalm no. 7 is also an appeal for protection, where David says " ... I take refuge in you; save and deliver me ... My shield is God".

Above the portals come the twenty-eight royal niches of the Gallery of Kings. The number 28 has different connotation in both astronomy and theology; in addition to being the number of the kings in the Old Testament, it is the number of years in the Gregorian calendar cycle, the number of the days in lunar cycle, and it is the number of generations between King David to St. Joseph (husband of Mary).

Above the gallery of kings is the **Rose Window** (Fig. 9) flanked by two lateral windows, each of them is topped with a *Solar Wheel* or *Sun Cross*.<sup>9</sup> The Rose Windows are the ultimate expression of the medieval geometry and the ultimate representation of the *squaring of the circle* concept (Wagstaff, 1922). Typically, the West Rose Window depicts the Second Coming of Christ (Cowan, 2005). The Rose Window here, with its radiating pattern and its position inside the overlap of the above mentioned Vesica Piscis of the elevation, represents the *iris* inside the all seeing eye of God, with the image of Christ as the *pupil*.

The main theme of the West Rose Window here is astronomy and life cycles, featuring scenes of Zodiacs and Labors of the Months,<sup>10</sup> the twelve months of the year, the four seasons, etc.

Sacrobosco's *De Sphaera mundi* (On the Sphere of the World) is a popular source of the medieval cosmology. A diagram in this reference depicts the *Hierarchies of the Universe*, where the outer circle, the Empyrian, is the highest heavenly realm (the Seat of God), while the sub-lunary realm (the Earth) is represented at the center as an actual *rose* (Edgerton, 1991). The translunary world is divided into nine spheres, the empyrium, the stars, and the seven planets with nine ranks of angels.

The resemblance between this diagram and the geometry of this Rose Window, with the rose at the middle, the nine circular divisions around it, and the 24 medallions in the outer ring, is inescapable. Concentric circles are also symbol of union and continuity (Barabanov, 2002). The

whole concept of *Sphaera mundi* refers to Heaven and Earth as one unit.

Immediately over the division containing the Rose Window, is the **Galerie des Chimères**; an open trefoil arcaded screen, surmounted by a pierced cornice, behind which rise the two towers. On the top of it are the famous grotesque figural **gargoyles**, which were thought to have the aptitude to ward off evil (Benton, 1997). The interface between the sacrum and the profanum is facilitated by formal separation, as the saints are arranged around portals and windows and do not keep grotesque company.

Lastly, the two lofty and massive towers rise behind the Galerie des Chimères. In general, the number two is known to be the number of revelation (Hopper, 1938). Everything in nature is dual and can only be known in contrast with its opposite. The **Twin Towers** of the Gothic cathedrals are said to be standing for the sun and the moon,<sup>11</sup> with their chain of associations: day/night, warm/cold, light/dark that denote all the *pairs of opposites*. It is also a symbol of the duality in the human body, where the person raises his two hands in the prayer.

#### 4.4. Side and rear fronts

The extended side elevations call forth an image of rhythmic and festive processions of people. In medieval occultism, the arch was seen as a horizontal metaphor, mediating between two parallel pillars (Barabanov, 2002). Pointed arch has another earthly metaphor, because it re-directs pressure toward the ground, rather than outward as in the round arch.

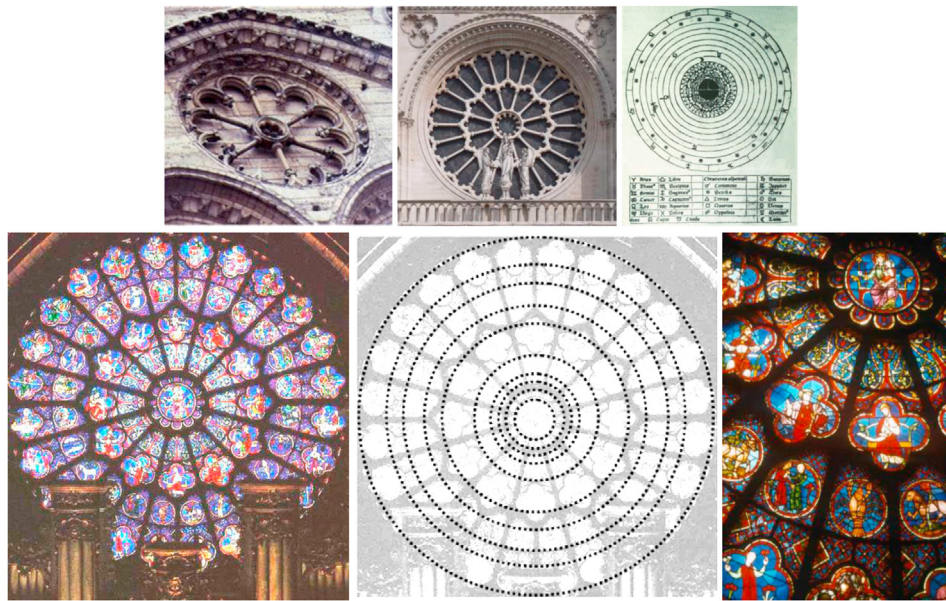
The horizontal line multiplied in the rows of arches is supplemented by the powerful continuous horizontal line of the roof. The strong verticality of the **portal's bay**, multiplied by the pointedness of the arches, balances this composition, symbolizes progress and ascension, and adds up to the image of the union of the upper and nether worlds (the whole composition recalls the Lord surrounded by his people). On each side of the portal are two blind arches. These, as well as the portal, all have pointed canopies. The relationship between number five with both protection and appealing to God had been already explained, where the pointedness of the canopies refer heavenwards. Above the five arches is the screen of arcade of lights, with nine

<sup>9</sup> A symbol of sun that symbolizes the marking of cross-quarter days (midpoints between seasons) along with the four spokes of the equinoxes and solstices.

<sup>10</sup> Like in the left portal.

<sup>11</sup> In Chartres cathedral the two towers, which are not twin towers, are capped with iron plates of sun and moon.





**Fig. 9** West Rose Window: (above, left to right) The Solar Wheel, the window from outside, Sacrobosco's *Hierarchies of the Universe* (Holybush, 1490), (below) the window from inside with its nine divisions and detail of the window.

arches (number of translunary world) topped by the Rose Window.

Here, again, the obvious correspondence between the scheme of the portal's bay and that of the ground plan (although the dimensions are different) is undeniable. The geometry of the pentagram and the Vesica Piscis in the portal are also easy to recognize (Fig. 10). The two portals are dedicated to Christian martyrs; St. Stephen, and St. Theophilus. The angle at the base of the triangular gables at the top is also  $52^\circ$  (the Pyramid) (Hale, 1995).

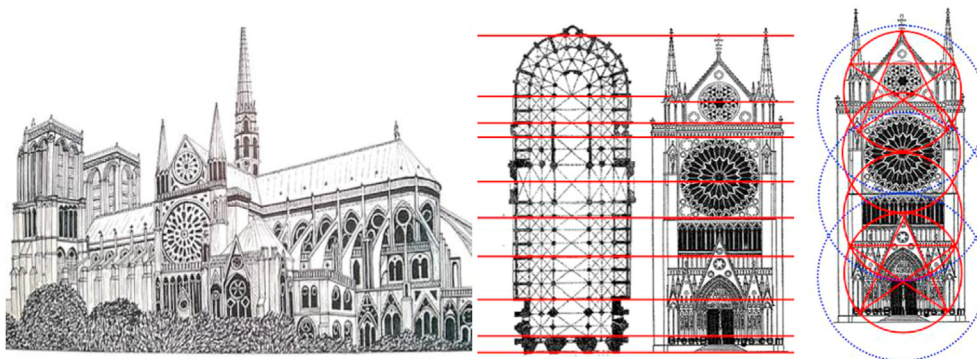
The most notable feature of each of the side fronts is the vast **Rose Window**. They both represent triumphant Christ surrounded by all his witnesses from both New and Old Testaments. Unlike all the other elements in the three fronts, the Rose Window is the only element that is not repeated, i.e., there is only one at each front, but there are three of them in the building as a whole; a composition that recalls the Trinity (one in three and three in one).

The geometry of these windows is one of the strongest applications of *squaring the circle*. As in the West Rose

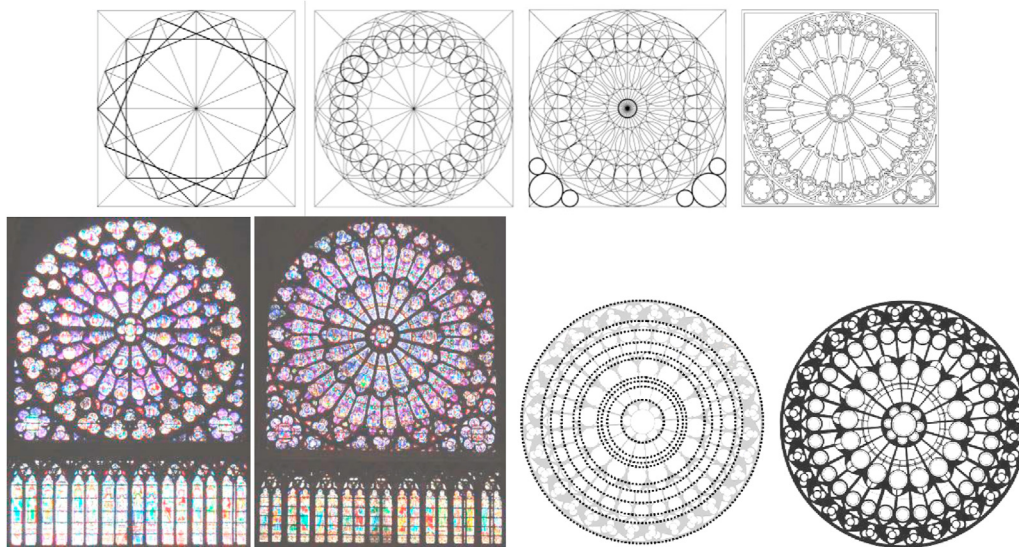
Window, the design of the side Rose Windows recalls that of the *Hierarchies of the Universe* (Fig. 11). It looks also like a radiating star; symbol of guidance.

The two windows feature number four and its multiples. The increase in numbers of subdivisions/segments (12–24–48) of the Rose Windows is an indication to infinity and everlasting growth. Each of the two windows consists of an overall 84 panels (DiLonardo, 2019). 84 is the result of  $7 \times 12$  (the number of prayers per day  $\times$  the number of God's people). Psalm No. 84 is also the one that says: "How lovely is your dwelling place .... Blessed are those who dwell in your house; they are ever praising you".

The northern Rose Window is dedicated to the New Testament. The first circle at the middle of it is surrounded by eight petals, or segments with multiplications of this number in the bigger circles. Number eight in St Augustine writings signifies the "eighth day"; the resurrection, or the life "made eternal", as well as being the number of survival (the people in the ark of Noah) (Augustine, 1972). It is also related to the Octagon, i.e., the Sacred Cut.



**Fig. 10** (from left to right) Side elevation (D'Agincourt, 1823), the correspondence between the plan and the side portals, and geometric analyses of the side portal.



**Fig. 11** (above) Ad Quadratum, Vesica Piscis, and the Circle of Unity in the geometry of the Rose Windows, (below, from left to right) the south Rose Window, the north Rose Window, and the nine concentric circles of the *Hierarchies of the Universe* in their geometry.

The main theme of southern Rose Window is the *Old Testament*. The central circle is also surrounded by eight (unequal) petals, but the geometry of the rest of the circular field shows a twelve-fold division. The nine-fold concentric divisions are also easy to recognize.

Below the Rose Window are 16 lancets, where the Heavenly Court is represented by sixteen prophets ( $4 \times 4$ ) and the four evangelists.

In the rear front, the balance between vertical and horizontal lines in the other fronts gives way to progression and ascension in their best representation. The spectacular protruding flying buttresses are symbolically linked to the giant oars of the Ark of Noah. They are arranged in two groups of seven buttresses 50 ft. ( $7 \times 7 + 1$ ) wide from each other. Seven is the number of prayers, while fifty is the number of jubilee, referring to deliverance and restoration following seven cycles of seven years (Hopper, 1938). The first level consists of a row of pointed arches surmounted by pointed triangular gables. These are then surrounded by the flying buttresses with their sloping upper webs like arms raised heavenwards, where the small pinnacles at the top of their vertical posts are like arrows pointing heavenwards. This whole composition is then topped by a pointed conical roof carrying the cross.

Gothic cathedrals are generally ornamented with liberated lines that strive for infinity. Worringer calls these lines *Northern Lines*, emphasizing that the suggestion of organic growth is these lines is unavoidable (Worringer, 1920). The little nodules that run the length of the ribs and the spire also suggest growth in the form of buds. These, as well as the repetitive openings and patterns in the three fronts and the spire are all good applications of *Fractals* (Joye, 2007).

#### 4.5. The towers and the spire

The towers and the spire are vertical elements that symbolize the aspirations to be united with God, and the

cathedral's contact with Heaven. They are also *the masts of the Ark*.

The cross, in general, is a symbol of union (Chevalier and Gheerbrant, 1988); the spire rising from the ending crossing of the vaults is therefore the *Axis Mundi* or world axis that connects Earth and Heaven (Barabanov, 2002). The original spire of the cathedral, which was a bell tower, was replaced by another one in 1792, which is not a bell tower. The bell tower is not only a visual symbol of ascension, but also a vocal symbol that is closely related to prayers. It is a voice that spreads and rises up to the Heaven, while in the same time it summons the people on Earth to recite the Lord's Prayer.

The spire is the only element in the cathedral that is not repeated at all; it is only *One*. One is the *Monad*<sup>12</sup>; a point of unity from which all numbers flow forth, and therefore all geometric forms. The *point*, at the apex of the spire and the pinnacles all over the roof, is reference to the absolute reality (Divine), a place of condensing and implementation; it is a symbol of creation and vanish - the beginning and the end (Barabanov, 2002).

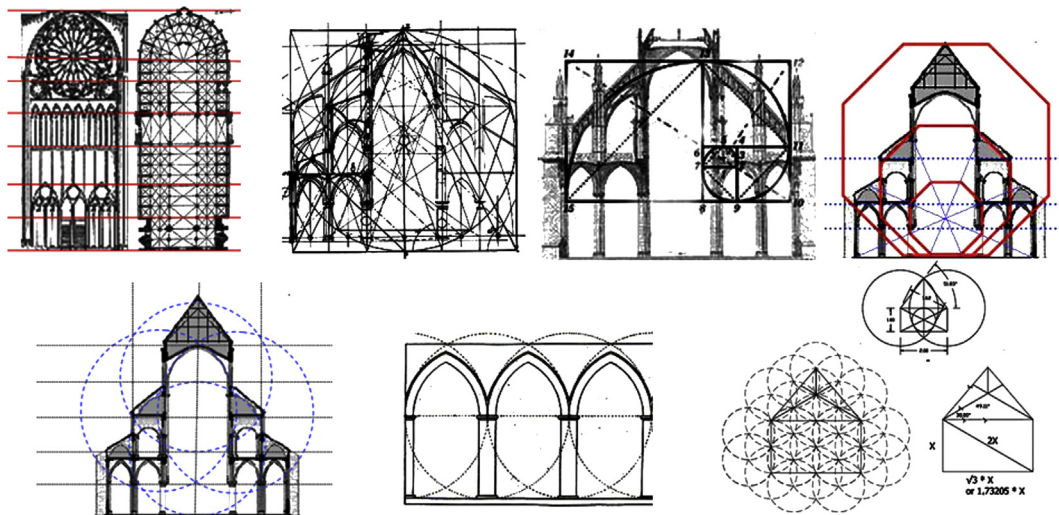
The spire here is a cone that is elongated to reach the sky. The cone, as a geometric body, unites the symbolism of a triangle, circle and center. It is related to the evolution from material to spiritual, to an image of progressive spirituality of the world (Barabanov, 2002).

#### 4.6. Interiors

Armed with geometrical logic, Gothic architects and craftsmen designed not only forms and structures, but they crafted cavernous interiors that aimed to reach the sky,

<sup>12</sup> According to the Pythagoreans, it was a term for Divinity (the first being), or the totality of all beings, *Monad* being the source or the *One* without division.





**Fig. 12** Interior: (left to right) the correspondence between the plan and the side portals from inside, Ad Quadratum, Golden Section in Viollet-le-Duc's cross-section at the nave, and the sacred cut in the geometry of the chancel (below) Vesica Piscis in the geometry of the chancel and the pointed arches, and the Circle of Unity in the geometry of medieval framing (Based on [De l'Orm \(1567\)](#)).

diminishing the human scale and stressing the insignificance of man against God's loftiness.

In contrast to classical architecture, the function of columns in Gothic cathedral is no longer merely to bear weight, but to symbolically draw the soul up into the Heavens. They do not come to a definite end (i.e., an architrave), but continue up until they meet to form a pointed arch or a vaulted roof ([Bruzelius, 1987](#)). Vertical lines are emphasized, giving an impression of going up to infinity, while the pointedness of the arches and the vaults gives further impression of continuity and pointing heavenwards.

The height of the nave was divided into three divisions: arcade, clerestory and triforium. Three is the number of the Divinity revealing itself (Holy Trinity) ([Horn and Born, 1975](#)). Walls are adorned by three bands of double lancet in horizontal repetition that gives an impression of endless multiplication and look like procession of congregation, who fill the cathedral even when there is no actual congregation in it. There is no access to the cathedral's upper levels, obliging visitors to remain earthbound, gazing upward to the unreachable *heavenly light*.

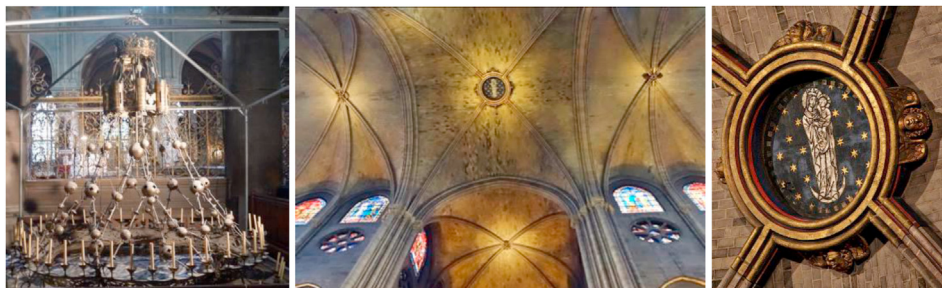
Cathedrals were often compared to Noah's Ark; the word Nave (Latin *Navis*) itself means a ship. Noah's Ark stood as

the symbol of the house protected by God. The geometry of arches ([Fig. 12](#)) is also based on the Vesica Piscis (fish) in another reference to *sailing*.

The three Rose Windows are the interior's most outstanding feature. The labyrinthine *squaring the circle* in their geometry represents Heavens around the Earth, where the radiant light recalls the eye of the Divine ceaselessly watching over his people.

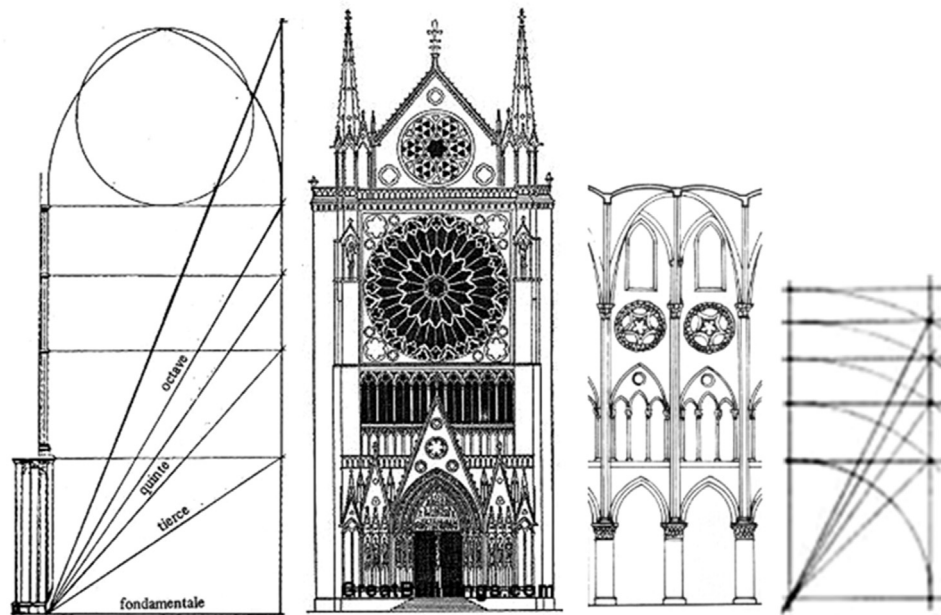
The twelve supporting columns for the ambulatory and choir are manifest of Abbot Suger's statement "we are building spiritually upon the foundation of Apostles" ([Wilson, 2009](#)). All around the sides of the church are niches containing statues of saints. Along the walls separating the central nave from the side isles are series of beautiful bas-reliefs that were meant to turn the walls into *stone bible*. There is also no shortage of the *secular* scenes of life, nor were Old Testament figures lacking, so that the entire human race since the earliest times is represented in the cathedral.

There is no stairs at the entrance level, but some stairs elevate the choir and the chevet, and above them further steps elevate the High Altar, raising the holiest part of the cathedral closer to the rising sun. A large isolating wall is installed around the choir. This wall is home to sculptures



**Fig. 13** The Crown of Light and detail of the medallions from which it used to hang.





**Fig. 14** Musical Notes of 5:3 (Golden Ratio) in the design of the side portal and the interior of the chancel in Notre Dame (based on De Honnecourt, 1953) and the square-root rectangles of  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{4}$  und  $\sqrt{5}$ , in the design of the bays.

that depict, from north to south (left to right), the life of Jesus, his crucifix, and his resurrection. At the East End, the apse has large clerestory windows that indicate the entry of spiritual light into the world.

Robert Willis in his book *On the Construction of Vaults of the Middle Ages*, overviews some Gothic vaulting techniques as originally illustrated by Philibert De l'Orme, where he explains how medieval builders used revolving circles, or Vesica Piscis, as invisible working lines to geometrize Gothic vaulting, producing the symbolic figures of the *Flower of Life* or *Circle of Unity*. These grids, in addition to the Fibonacci sequence nested in its structure (Fig. 2), are able to keep repeating themselves to infinity inward and outward in a continuous geometrical progression, denoting wholeness, infinity and protection (Ramzy, 2015). Fig. 12 demonstrates some further applications of *squaring the circle* in the geometry of the interior, i.e. Golden Section, Vesica Piscis, and Sacred Cut.

Light is one of the most important elements of Gothic cathedrals. At this time, light was seen as a mystical force; a medium that is somewhat intermediate between the earthly and heavenly. Pseudo Dionysius, who is also a Neo-Platonist, whose writings were highly appreciated by Abbot Suger, describes it as "the highest fleshly state that human sensibility could directly perceive is light, which is very nearly pure spirit" (Von Simson, 1956). Wim Swann explains the symbolism of light saying that: "Light, which could pass through glass without breaking it, was likened to the Word of God, Light of the Father that came to the world through the body of the Virgin" (Swann, 1969). The windows in the upper level are bigger and directly overlooking the nave, where the other windows are partially separated from the nave by the arcades of the aisles, so that the higher one looks, the more light one sees, as if getting closer to Heaven.

The *Crown of Light*<sup>13</sup> is a great chandelier that used to hang from the point of the transept crossing. This point was marked with a big medallion (Fig. 13), from which the light comes downwards to the earthy domain through the Crown of Light and the spire (Axis Mundi) rises up to reach the heavenly domain.

For St. Augustine music and architecture were sisters: "the architecture reflects the eternal harmony and music is their echo" (Bandmann, 1951), where Otto von Simson referred to the harmonic special conception of Gothic cathedrals as "music turned into stone" (Von Simson, 1956). The notebook of Villard de Honnecourt (published in 1240) shows the musical ratios in the proportions of the nave elevation of Reims cathedral (Jencks, 2013). The use of both musical ratios and the root rectangles in the design of Notre Dame de Paris is similarly shown in (Fig. 14).

## 5. Discussion

In medieval tradition, Christians were asked to "pray without ceasing" as they struggle against spiritual, cosmic forces that never sleep (Brümmer, 2008). To fulfill this extreme demand, Origen encouraged them to interpret acts of righteousness and leniency as acts of prayer, saying that: "He prays without ceasing who combines his prayer with necessary works, ...Only in this way the whole life of someone is to become one mighty integrated prayer" (Origen and Jay, 1954).

On another side, Robert Scott says that at this time, it was not taken for granted that God would inhabit any church building; Bishops strove to emulate "the image of God's kingdom on earth" to make their cathedrals entitled

<sup>13</sup> Now restored in the north ambulatory.

to God's presence (Scott, 2003). It is expected then that they tried to include the main feature of this kingdom, which is ceaseless hymns, in their buildings.

The early writings of the Christian Church bear witness to a prayer tradition that is rich in eschatological symbols (Brümmer, 2008), so were the traditions of medieval masons in possession of "great secrets" (Paine, 1818); one of them is their symbolic tradition, or what Richard Utz calls the "Concept Cathedral", in reference to the hidden "meaning" included in these cathedrals. At this time, where architecture was seen as a *Divine Work*, and where in the Regius Poem; the oldest Masonic constitution (ca. 1390), the words *pray* and *prayers* are repeated 14 times, in one of them it says that during their work masons should: "make no noise, but always to be in thy prayer" (Hughan, 1872), it was expected then that they tried to consider Origen's advice by turning their work into a prayer.

The main prayer book used in medieval Europe was "The Book of Hours", which was divided into seven sections, or *Hours*. A typical Book of Hours contains (Nassar, 1938):

- The seven Hours (the Hours of the Cross): psalms, prayers, and other readings
- Excerpts from the four gospels
- The Little Office of the Blessed Virgin Mary
- A Litany of Saints
- An Office for the Dead
- Various other prayers

Morning prayers were focused on the *Risen Messiah*, evening prayers on the continual need for forgiveness and protection from the cosmic and evil forces, and night prayers on the *Second Coming of the Messiah* (Taft, 1986).

The sign system included in the cathedral (as suggested in the previous discussions) is illustrated in Fig. 15, summarized and organized into syntagms in Table 1. In a direct reading concerning the obvious functions of each of the architectural elements included in this table, most of them would be found functionless. The meaning that they connote is the real issue behind their existence.

This sign system includes all three types of signs (iconic, symbolic, and indexial). However they could be grouped in 5 main groups (syntagms): (1) starting the prayer; (2) the theme of the prayer: protection and unity with Heaven; (3) the symbolic congregation, who are always present in the cathedral: apostles, saints, kings and all the Heavenly Court; (4) the duration of the prayer: day and night, seven days per week, four weeks per month, twelve month and from seasons per year; life cycles; from the introduction of the heavenly light (Jesus Christ) into the world, till his Second Coming ... etc.; (5) ending the prayer and sending it heavenwards.

The sign-system included in the plan (Fig. 15a) begins with *squaring the circle*, to unify the building with the heavenly domain. The masons then put the *foundation* for starting a prayer/hymn: the orientation to the east, the sign of the cross, the letters of language, the musical notes, and the people of God with the 12 apostles among

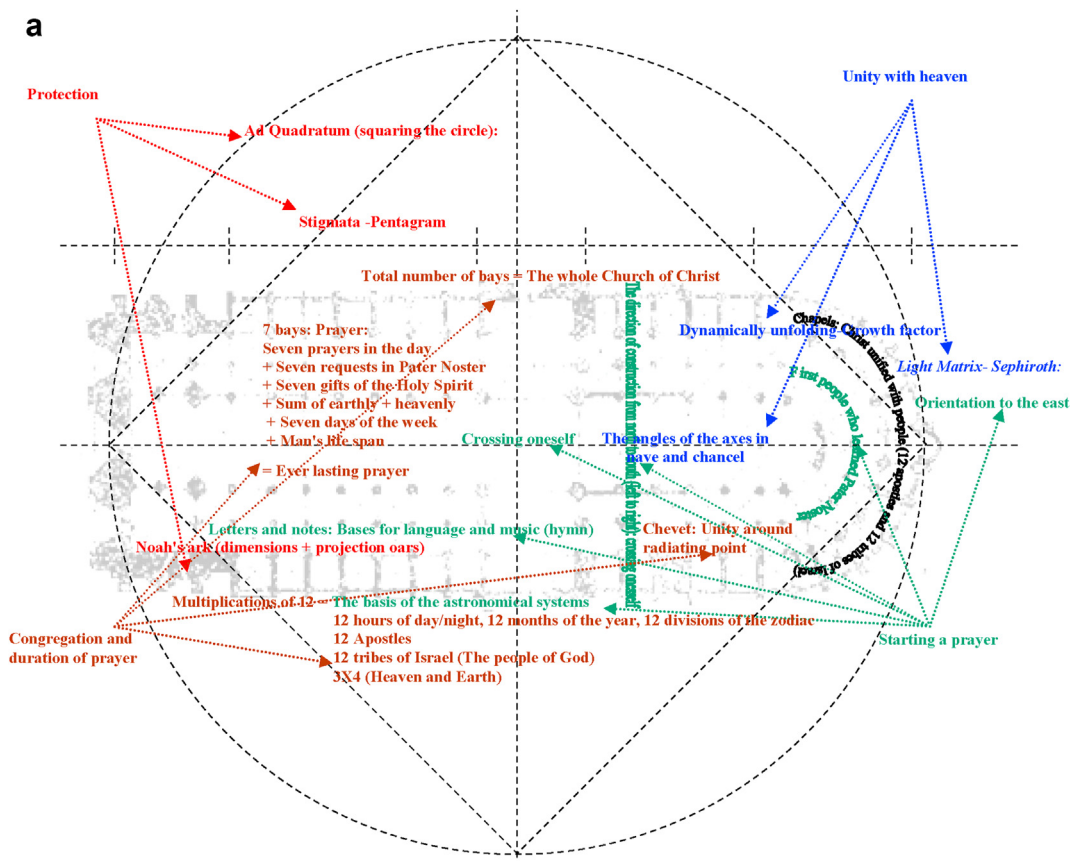


Fig. 15 a: The sign-system in the plan. b: The sign-system in the façade. c: The sign-system in the interior.

them as *the foundations*. They also put the basis for the unlimited *growth* of this structure heavenwards by using "dynamically unfolding geometrical operations" that provides "a quasi-random growth factor" (Bork, R., 2014).

The construction process was then divided into seven stages, as in the seven prayers of the day, with multiple appearance of this number in several other parts of the building: The nave is divided into 7 bays in both directions; four groups of seven flying buttresses (2 groups on the sides

b

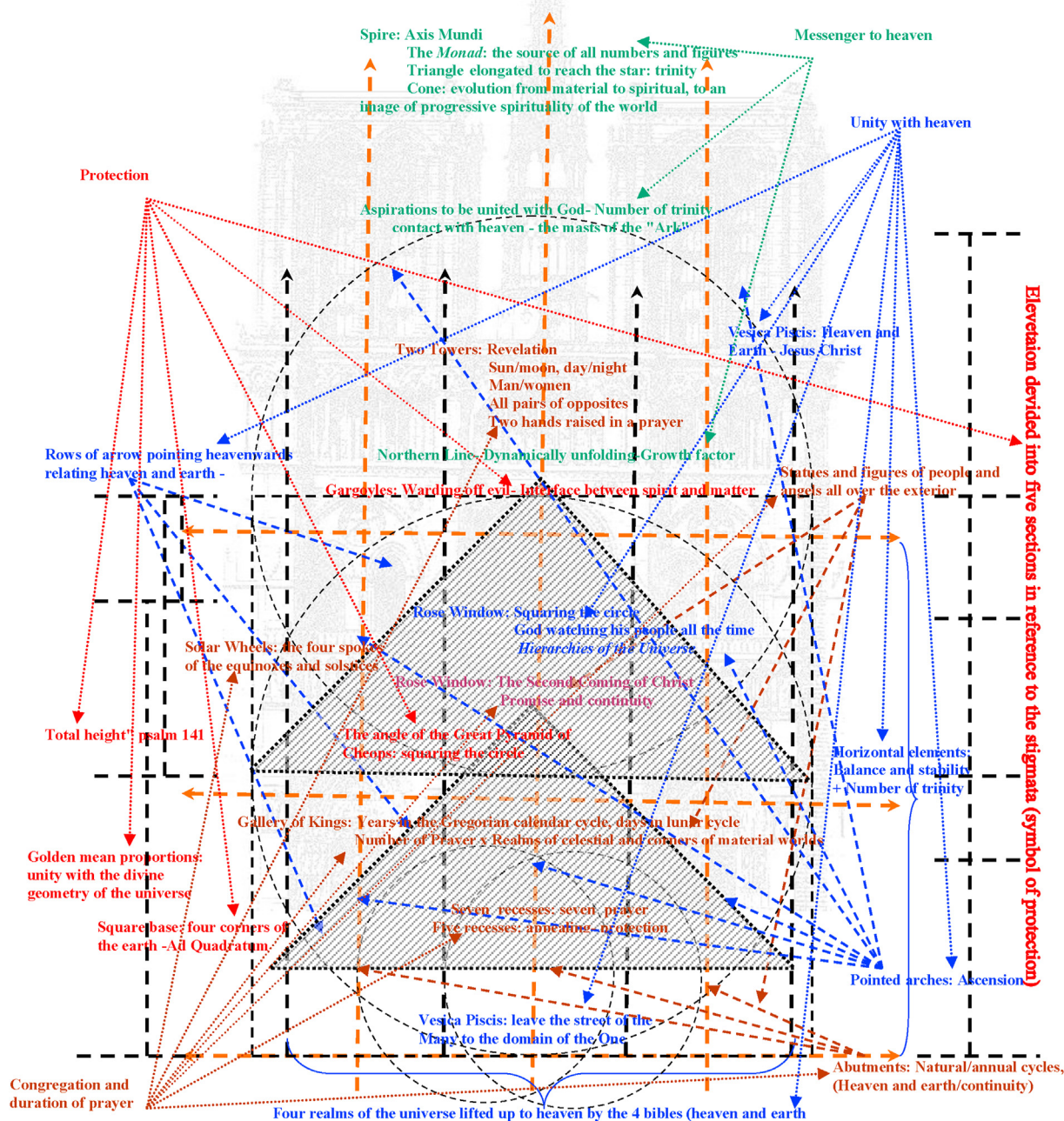


Fig. 15 (continued).



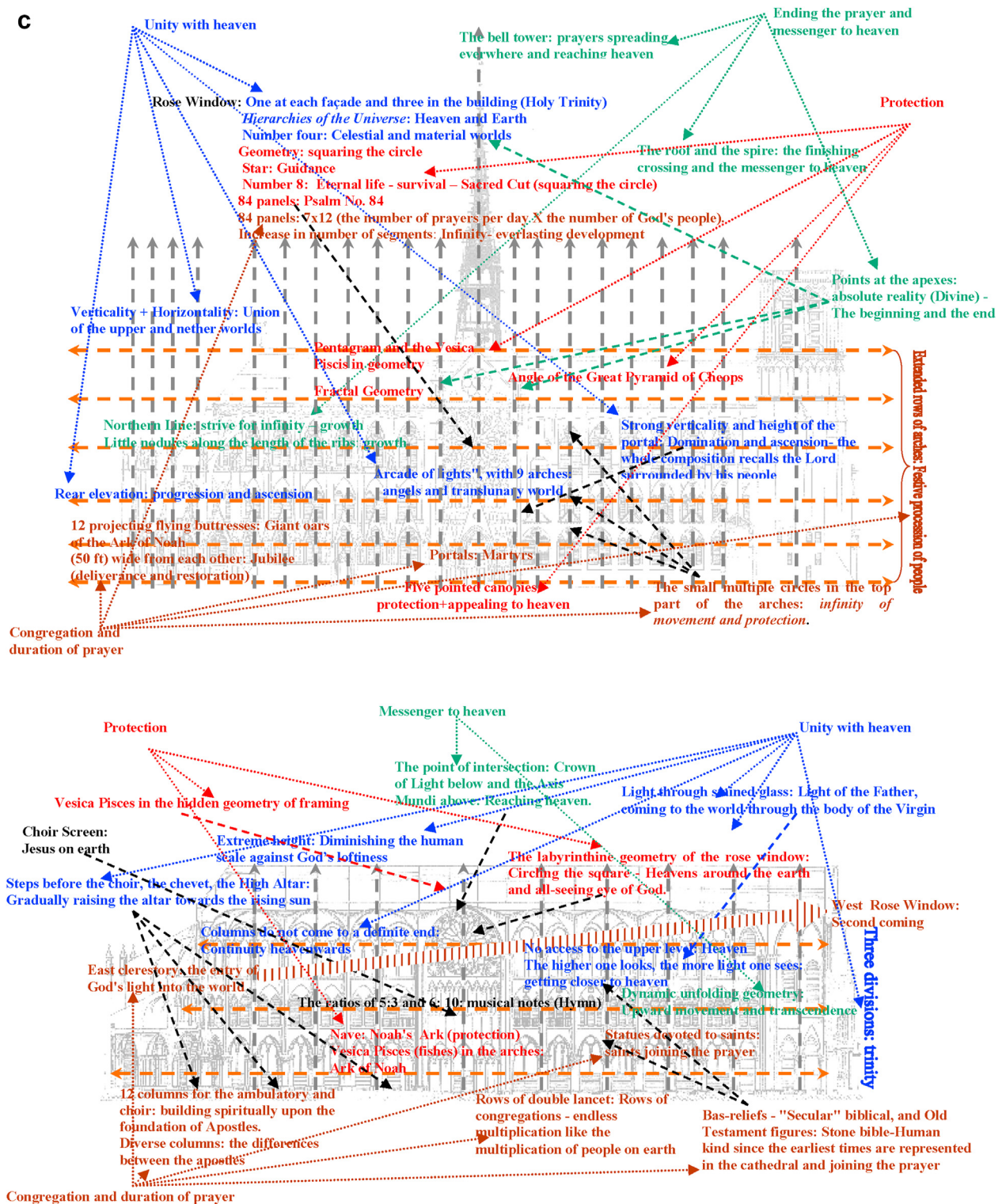


Fig. 15 (continued).

of the chancel and 2 groups on the sides of the nave); seven recesses in the main archivolts, etc.

Geometry, which was considered then as the sacred language of universe, played an important role in creating this *hymn* (Fig. 15b, c). Different numerical and geometric symbols were included in the design of the cathedral,

bearing mainly the concept of *squaring the circle* as means for *protection* and *unity with heaven*, as discussed in Section 3, where figurative and visual symbols were used to refer to ascension and aspiration heavenwards. These were the symbols that contained the "theme of the prayer", which is an appeal to get protection for the building from

**Table 1** Semiotic analyses of the elements of the Prayer/Hymn included in the architectural language of Notre Dame de Paris

Signifier	Signified	Mode of sign	Denotation/ connotation	Paradigms/syntagms			Metaphor/ Metonymy
1. Ad Quadratum: Circular base 4 stones for corners	Heaven	Symbolic	Connotation	Paradigm	Syntagm: Squaring the circle	Syntagm: Starting a prayer/hymn	Metaphor
	Four corners of the world	Symbolic	Denotation	Paradigm			Metonymy
2. Orientation of the building	Starting a prayer	Indexial	Connotation	Paradigm			Metaphor
3. Cruciform ground plan - direction of construction from north to south	Crossing oneself at the beginning of prayer	Iconic	Denotation	Paradigm			Metonymy
4. 12 stones for the foundations Stones washed with water and wine	the first people who learned the prayer	Indexial	Connotation	Paradigm	Syntagm: Apostles		Metonymy
	The Last Supper (apostles)	Indexial	Denotation	Paradigm			Metonymy
5. Construction began at the East 12 columns for the ambulatory and choir:	Looking east to start a prayer	Indexial	Connotation				Metonymy
	The basis of the astronomical systems Building spiritually upon the foundation of Apostles.	Indexial	Denotation	Paradigm	Syntagm: Basis of language and music		Metonymy
	8 columns on each side of the Nave	Indexial	Connotation	Paradigm			Metonymy
Total number of piers is 26	Number of letters in Latin.	Iconic	Connotation				Metonymy
6. Construction process in 7 stages The nave is divided into 7 bays in both directions Four groups of seven flying buttresses (2 groups on the sides of the chancel and 2 groups on the side of the nave)	7 days per week/human life span + Prayer/ ritual (an everlasting prayer)	Indexial	Denotation	Paradigm		Syntagm 7 prayers per day	Metaphor
	4 weeks per month, four seasons in the year x 7 prayers a day (Rituals and prayers support of the building)	Indexial	Connotation	Paradigm			Metaphor

7.	Seven recesses in the Prayers main archivolts 84 panels	Indexial	Denotation	Paradigm			
	7 × 12 (the number of prayers per day X the number of God's people). Psalm No. 84	Indexial	Connotation	Paradigm			
	Dynamic unfolding geometry	Growth factor (Upward movement and transcendence heavenwards)	Symbolic	Connotation	Paradigm : Ascension		Metaphor
	Extreme height	Diminishing the human scale against God's loftiness	Indexial	Denotation	Paradigm		Metonymy
	Steps before the choir, the chevet, and the High Altar:	Gradually raising towards the rising sun	Indexial	Connotation	Paradigm		Metaphor
	Vertical elements (spire and towers)	Upwards aspiration + Number of trinity	Iconic	Denotation	Paradigm		Metaphor
	Vertical elements within the horizontal elements	Rows of arrows pointing heavenwards	Iconic	Denotation	Paradigm		Metaphor
	Pointed arches	Ascension	Iconic	Denotation	Paradigm Syntagm	Syntagm: Unity with Heaven	Metonymy
	4 powerful buttresses and 4 sections of elevation	4 realms of the universe and 4 corners of Earth lifted up to Heaven by the 4 bibles	Indexial	Connotation	Paradigm		Metonymy
	Strong verticality and height of the portal	Domination and ascension	Iconic	Denotation	Paradigm		Metonymy
	Geometry of the rear elevation	Progression and ascension	Iconic	Denotation	Paradigm		Metonymy
	Columns do	Continuity	Symbolic	Denotation	Paradigm		Metaphor

(continued on next page)



Table 1 (continued)

Signifier	Signified	Mode of sign	Denotation/ connotation	Paradigms/syntagms	Metaphor/ Metonymy
not come to a definite end	heavenwards				
8. The axes of the nave and choir	Connecting Heaven to Earth	Indexial	Connotation	Paradigm Syntagm: Connection	Metonymy
The dimensions of plan contained in the elevations and sections	Continuity and unity between horizontal and vertical (Heaven and Earth)	Indexial	Connotation	Paradigm	Metaphor
Light Matrix - <i>Sephiroth</i>	Spirit ( <i>light</i> ) and Matter ( <i>man</i> ) come together - Divine power	Symbolic	Connotation	Paradigm	Metaphor
The arcade of lights (9 arches)	Translunary world	Indexia	Connotation	Paradigm	Metonymy
No access to the upper levels	Heaven	Indexial	Denotation	Paradigm	Metaphor
Three divisions of the height of the nave	Trinity	Indexial	Denotation	Paradigm	
Three Rose windows, one at each side:	Trinity	Indexial	Denotation	Paradigm	
Rose window (one at each façade and three in the building)	Trinity: one in three	Indexial	Connotation	Paradigm	Metaphor
Number four in the Rose Window	Celestial and material worlds	Indexial	Connotation	Paradigm	Metaphor
Verticality of the pointed arches of side	Union of the upper and nether worlds (the whole	Iconic	Denotation	Paradigm	Metonymy
portals + Horizontal rows of arches & roof	composition recalls the Lord surrounded by his people)				
The point of intersection of the vaults	Crown of Light below and the <i>Axis Mundi</i> above: Heaven (light) reaching Earth, and Earth (world axis) reaching Heaven	Symbolic	Denotation	Paradigm	Metonymy
Light through stained glass:	Light of the Father, coming to the world through the body of the Virgin	Indexial	Connotation	Paradigm	Metaphor

	The higher one looks, the more light one sees:	Getting closer to Heaven	Indexial	Denotation	Paradigm		Metaphor
9.	The Dimensions of Noah's ark	Protection and salvation	Indexial	Denotation	Paradigm	Syntagm: Noah's Ark	Metonymy
	Nave + Vesica Pisces (fishes) in arches	Noah's Ark (protection)	Symbolic	Connotation	Paradigm		Metaphor
	12 projecting flying buttresses	The Giant oars of the Ark (apostles)	Iconic	Connotation	Paradigm		Metaphor
	Five divisions of the plan and elevation	Stigmata (Christ-protection) Pentagram (man) 4 + 1 = the Earth + the Devine	Indexial	Connotation	Paradigm		Metaphor
	Fractal patterns	Geometrical progression - geometry of Nature	Indexia	Connotation	Paradigm		Metonymy
10.	Square base of the West Facade	Ad Quadratum- Stability	Iconic		Paradigm	Syntagm: Squaring the circle	Metaphor
	Geometry of the Rose Window	Ad Quadratum- Hierarchies of the Universe	Indexial	Connotation	Paradigm		Metonymy
	Golden Section	Squaring the circle- geometry of Nature	Symbolic	Connotation	Paradigm		Metonymy
	<i>Ad Quadratum:</i> Vesica Pisces, sacred cut, Circle of Unity and pentagram	Squaring the circle	Symbolic	Connotation	Paradigm		Metaphor
	The angle of the Great Pyramid of Cheops in the elevation and the gable	Squaring the circle	Symbolic	Connotation	Paradigm		Metaphor
	Rose Window	Squaring the circle	Indexial	Connotation	Paradigm		Metaphor
	Number eight in the Rose Window	All seeing eye of God Eternal life - <i>Sacred Cut</i> (squaring the circle)	Symbolic Indexial	Connotation			Metaphor
	The labyrinthine geometry of the rose window	Squaring the circle Heavens around the Earth and God watching the lives of creations.	Iconic	Connotation	Paradigm		Metaphor

(continued on next page)

Table 1 (continued)

Signifier	Signified	Mode of sign	Denotation/ connotation	Paradigms/syntagms		Metaphor/ Metonymy
Star	Guidance	Iconic	Denotation	Paradigm		Metonymy
Total height of the facade	Psalm 141 (protection and savior)	Indexial	Connotation	Paradigm		Metonymy
12. Gargoyles	Warding off evil	Iconic	Connotation	Paradigm	Syn. Appeal	Metonymy
Five successive archivolt (lateral portals)	Appealing to god	Indexial	Denotation	Paradigm		Metaphor
Five pointed canopies on the side portals	Appealing sent heavenwards	Indexial	Connotation	Paradigm		Metaphor
Two Towers	Two hands raised in a prayer	Symbolic	Denotation	Paradigm		Metaphor
13. The ratios of 5:3 and 6: 10:	Musical notes	Indexial	Connotation	Paradigm	Hymn	Metonymy
14. Total number of bays in plan = 153	The whole Church of Christ	Indexial	Connotation	Paradigm	Syntagm: Congregation	Metaphor
Statues of people and angels all over the exterior in position of devoutness and reverence	Everyone in Heaven and Earth	Iconic	Denotation	Paradigm		Metonymy
Extended rows of arches	Festive processions of people	Iconic	Denotation	Paradigm		Metaphor
Chevet: 12 chapels and an apse	Unity around radiating point Christ unified with people from Old and New testaments (apostles- tribes of Israel)	Symbolic	Denotation	Paradigm		Metonymy
Rows of double lancet:	Rows of congregations- Endless multiplication of people on earth	Iconic	Denotation	Paradigm		Metaphor
"Secular" scenes of life and Old Testament figures	Human kind since the earliest times are represented in the cathedral	Iconic	Denotation	Paradigm		Metaphor



Statues of saints on the sides of the nave	Saints joining the prayer	Iconic	Denotation	Paradigm Litany of Saints	Syn. The Book of Hours	Metonymy
Side portals	Christian martyrs	Iconic	Denotation	Paradigm		Metonymy
Lateral portals on West Façade	Tympanums: Virgin Mary and Heavenly Court	Iconic	Denotation	Paradigm Office of the Virgin		Metonymy
Choir screen	Jesus on earth (stone bible)	Iconic	Denotation	Paradigm Stone bible		Metaphor
Bas-reliefs illustrating the life of Jesus	Stone bible	Iconic	Denotation	Paradigm		Metonymy
Scenes on the stained glass	Old and New Testaments	Iconic	Denotation	Paradigm		
Gallery of Kings:	Generations from David to St. Joseph.	Indexial	Connotation	Paradigm Office for the Dead		Metonymy
Lateral portals on West Façade	Kings and bishops of France	Iconic	Denotation	Paradigm		
15. East clerestory	The entry of spiritual light into the world	Indexial	Connotation	Paradigm	Syntagm: Extent	Syntagm: Duration of the prayer
West Rose Window	The Second Coming	Indexial	Connotation	Paradigm		
16. Four groups of seven flying buttresses	4 weeks per month, four seasons per year x 7 days per week - human life span +	Indexial	Connotation	Paradigm	Syntagm: Continuity (day and night, 4 weeks per month, 12 months a year, etc.	
Abutments of the Portals	Natural/annual cycles	Iconic	Denotation	Paradigm		Metonymy
Gallery of Kings (28 kings)	Years in the Gregorian calendar cycle, days in lunar cycle	Indexial	Connotation	Paradigm		Metonymy
Solar Wheels	Solar cycle	Symbolic	Connotation	Paradigm		Metaphor
Two Towers:	Sun/moon, day/night- Man/women -All pairs of opposites	Indexial	Denotation	Paradigm		Metaphor
Multiplications of the Rose Windows	12 months of the year, the four seasons Everlasting increasing	Indexial	Connotation	Paradigm		Metonymy
Increase in number of segments (50 ft) wide from each other.	Jubilee (deliverance and restoration)	Indexial	Connotation	Paradigm		Metonymy

(continued on next page)

Table 1 (continued)

Signifier	Signified	Mode of sign	Denotation/ connotation	Paradigms/syntagms	Metaphor/ Metonymy
17. West Façade (last part to be built)	Last Judgment	Iconic	Denotation	Paradigm	Syntagm Ending prayer
The roof and the spire:	The finishing crossing a	Iconic	Denotation	Paradigm	Metonymy
18. The vertical elements	Aspirations to contact with Heaven- masts of the "Ark"	Iconic	Denotation	Paradigm	Metaphor
The spire:	Axis Mundi - elongation to reach the sky - evolution from material to spiritual	Indexial – Iconic	Connotation	Paradigm	Syntagm: Messenger to Heaven
Northern Line	Strive for infinity-growth	Iconic	Connotation	Paradigm	Metonymy
Bell tower:	Prayers spreading everywhere and reaching Heaven	Indexial	Connotation	Paradigm	Metonymy
Little nodules along the ribs and spire	Eternal growth	Iconic	Denotation	Paradigm	Metaphor
Points at the apexes:	Absolute reality (Divine) The beginning and the end	Indexial	Denotation	Paradigm	Metonymy

evil forces and disasters and to unite it with the eternal world of Heaven.

Construction process began from the eastern part, where the *First Light* comes from the east clerestory in reference to Christ's First Coming, and where the life of Jesus Christ on Earth is the main theme for the scenes of the *Stone Bible*. The last thing to be built in the cathedral was the west side, the fundamental theme of which is the last judgment or the Second Coming, in reference to a prayer that last all along the period between the two *Comings*. It is notable here that this sequence copes exactly with the divisions of prayers throughout the day in the Book of the Hours; i.e. morning prayers for the Risen Messiah, evening prayers on forgiveness and protection, and night prayers on the Second Coming. The roof and the spire are the finishing crossing at the end of the prayer and the messenger that will lift it to Heaven. The musical ratios made the cathedral "built music" (Chesterton, 1920) and turned the *prayer* into a *hymn*.

Most of the signs in Notre Dame de Paris are Indexial, connotative, and metaphoric; they do not convey a direct denotation of the first order that is easy to understand by everyone, but rather an indirect connotations that are meant for a knowledgeable eye; the eye of the great mason of the universe. The sign system of the building consists of connotations of second-order that composes, as a group, a third order system of complete *sentences*, where single paradigms are gathered into meaningful syntagms. As a whole, the building introduces a full cultural picture of a *prayer*, as introduced in the medieval liturgies, i.e., the Book of the Hours.

The syntagmic arrangement of the signs in the building is very remarkable. Paradigms are arranged in two, and sometimes three, chains of signs that are interwoven into each other in a firm arrangement. In addition to the two main chains/syntagms, which represent the "theme of the prayer" or the Seven Hours, some other small syntagms are interwoven into the syntagm of the "congregation" to represent the rest of the elements of the Book of the Hours, i.e., Little Office of the Blessed Virgin Mary, Litany of Saints, Office for the Dead, etc.

## 6. Conclusion

In this study, semiotic tools were used to interpret the architectural message of the cathedral of Notre Dame de Paris. By decoding the *text* of this cathedral, semiotics had been introduced as a helpful tool for perceiving the meaning of an architectural text that was created according to a complex interplay of certain codes and conventions, of which people of the present time are not really aware. With semiotic tools, and according to a certain cultural code, which was in this case the writings of St. Augustine and his Neo-Platonic theological approach, the task of suggesting a reading the "secret language" of the medieval masons became available. In so doing, the paper presented a model for decoding architectural language in historical buildings.

The reading for the *text* included in the cathedral was found to represent a *Standing Hymn* that would continue as long as the building stands, sending appeals for protection

from evil forces and disasters to the Heaven. However, there is yet more than one text to be read in the cathedral. In addition to this text, which was *written* from east to west for the eye of the Divine, *another text*, among several other texts, was written for the eye of the worshipers, who read the building from west (the entrance) to east (the altar). It begins with the *Gates of Heaven*, from which one enters into what was seen as an earthly replica (Chesterton, 1920) for the heavenly Jerusalem.<sup>14</sup>

The interpretation of the text that medieval builders had woven in this building shows that, in building their *hymn*, the designer employed a complex system of formal expression, whose conventions governed the overall design of the building. Studies in this regard have the potential to enrich not only the scholarly discourse on medieval architecture, but also a larger and broader conversation about the issue of integrating a *meaning* in architectural formal expression.

The analysis in this paper shows that the highly sophisticated formal content of Gothic architecture still offers an alternative of inspiration for contemporary architectural practice, in much the same way it did in the 19th and early 20th centuries.

In addition to its spiritual dimension, this visual dialog, which directly addresses human's fundamental sensations, had certainly played a role in giving the Gothic cathedrals this pleasant effect as it answers the human need of *meaning* that Ernst Cassirer calls for. It is therefore still able to inspire contemporary architects an innovative approach to insert their own legacy and metaphors into their designs.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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<sup>14</sup> For this reason, the crypts of the earlier cathedrals were replaced by golden shrines.

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